Physics Letters B 794 (2019) 1–6

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Corrections to scattering processes due to minimal measurable length

Mir Faizal [a,b], S.E. Korenblit [c,e], A.V. Sinitskaya [c,d], Sudhaker Upadhyay [f]

[a] Department of Physics and Astronomy, University of Lethbridge, Lethbridge, AB T1K 3M4, Canada
[b] Irving K. Barber School of Arts and Sciences, University of British Columbia – Okanagan, 3333 University Way, Kelowna, British Columbia V1V 1V7, Canada
[c] Department of Physics, Irkutsk State University, 20 Gagarin blvd, Irkutsk 664003, Russia
[d] Irkutsk National Research Technical University, 83 Lermontov str., Irkutsk 664074, Russia
[e] Joint Institute for Nuclear Research, RU-141980, Dubna, Russia
[f] Department of Physics, K.L.S. College, Nawada-805110, Bihar, India

ARTICLE INFO

Article history:
Received 4 March 2019
Received in revised form 26 April 2019
Accepted 2 May 2019
Available online 6 May 2019
Editor: M. Cvetič

ABSTRACT

In this paper, we will analyze the short distance corrections to low energy scattering. They are produced because of an intrinsic extended structure of the background geometry of spacetime. It will be observed that the deformation produced by a minimal measurable length can have low energy consequences, if this extended structure occurs at a scale much larger than the Planck scale. We explicitly calculate short distance corrections to the Green function of the deformed Lippmann-Schwinger equation, and to the conserved currents for these processes. We then use them to analyze the pre-asymptotic corrections to the differential scattering flux at finite macroscopically small distances.

We do not have a complete theory of quantum gravity, however, there are various approaches to quantum gravity. It is expected from various different approaches to quantum gravity that the geometry of spacetime could be deformed by the existence of a minimal measurable length scale [5]. In fact, it is known that in string theory, the background geometry of spacetime gets deformed by the existence of a such minimal measurable length [1,2]. The reason is that the smallest probe available in string theory is the fundamental string, and so the spacetime cannot be probed below the string length scale [3,4]. In fact, it has been demonstrated that in perturbative string theory, the minimal measurable length l_{min} is related to the string length as $l_{min} = g_s^{1/4} l_s$ (where $l_s = \alpha'$ is the string length, and g_s is the string coupling constant). Even though non-perturbative effects can produce point like objects (such a $D0$-branes), it can be argued that a minimal length of the order of $l_{min} = l_s g_s^{1/3}$ is produced by these non-perturbative effects [5,6]. Such a minimal measurable length exists in string theory because the total energy of the quantized string depends on the winding number w and the excitation number n. Now under T-duality, as $\rho \to l_s^2/\rho$, we have $n \to w$. Thus, it is possible to argue using the T-duality that a description of string theory below and above l_s are the same, and so string theory contains a minimal measurable length scale [5]. It should be noted that an effective path integral of the center of mass of the string (for strings propagating in compactified extra dimensions) has been constructed, and T-duality has been used to demonstrate that such a system has a minimal length associated with it [7,8]. As the construction of double field theory has been motivated from T-duality [9,10], it is expected that such a minimal length will also exist in the double field theory [11].

It may be noted that even if the string theory does not turn to be the true theory of quantum gravity, the argument for the existence of a minimal measurable length in spacetime could still hold. As it can be argued, a minimal measurable length scale, at least of the order of Planck length, would exist in all approaches to quantum gravity. This is because any theory of quantum gravity has to produce consistent black hole physics, and the black hole physics can be used to prove the existence of a minimal measurable length of the order of Planck length. The reason is that the energy needed to probe spacetime below Planck length is less than the energy needed to produce a mini black hole region of spacetime [12,13]. As production of such a mini black hole would restrict our ability to probe this region of spacetime, so the black hole physics can also predicts the existence of a minimal measurable length. In fact, it has been demonstrated that an extended structure in the background geometry of spacetime also exists in the loop quantum gravity [14], and it is responsible for removing the big bang singularity. Furthermore, a minimal measurable

E-mail addresses: mirfaizalmir@googlemail.com (M. Faizal), korenb@ic.isu.ru (S.E. Korenblit), lasalvia@mail.ru (A.V. Sinitskaya), sudhakerupadhyay@gmail.com (S. Upadhyay).

https://doi.org/10.1016/j.physletb.2019.05.007

length exists even in Asymptotically Safe Gravity [15]. It has also been argued that such a minimal measurable length will exist in conformally quantized quantum gravity [16]. As it is possible than such a minimal measurable length exists in spacetime, it is important to study the consequences of the existence of such a minimal measurable length.

As it is possible for the string length scale to be several orders larger than the Planck scale [5], the minimal length scale can also be much larger than the Planck length. In fact, it is possible to argue that in most models of quantum gravity such a minimal length can be much larger than Planck length (and it would only be bounded by the present experimental data) [17,18]. It has been suggested that such a minimal measurable length much larger than Planck scale should have a measurable effects, which can be detected by performing more precise measurement of Landau levels and Lamb shift [19]. Actually, it has also been proposed that such a minimal measurable length will deform quantum systems, and this deformation can be detected experimentally using an opto-mechanical setup [20]. As this deformation can even be detected in precise measurement of low energy systems, it can also be detected in special future scattering experiments. So, it is important to consider the corrections to various scattering processes from such a minimal length. It has been suggested that the interaction between neutrons and a gravitational field can be measured using a gravitational spectrometer [21,22]. The deformation of such a system by the minimal measurable length, and its possible detection using such a gravitational spectrometer, has also been studied [23]. In fact, it has also been possible to obtain the corrections to quantum field theories and gauge theories (including standard model) from such a deformation [24–28]. Thus, it is important to analyze the effect this deformation will have on scattering processes. So, in this paper, we will analyze the modifications to a low energy scattering process by the existence of such an minimal measurable length in spacetime. This minimal measurable length acts as an extended structure in spacetime, and the scattering of an extended structures is very different from the scattering of point particles. However, if the extended structure exists at a very small scale, then at large scale these phenomena can be expressed as a scattering of point particles. The corrections to these phenomena will occur at intermediate scales, and those will be the corrections we will analyze in this paper. This analysis implies an accurate description of internal finite (macroscopically small) distance corrections to the scattering process itself, that will be done here.

It may be noted that it is possible for the deformation to occur at a scale much larger than Planck scale, and this scale would be bounded by the current experimental data [17–20,23]. However, a deformation at such a scale would have measurable consequences for low energy phenomena, and this will hold for accurate measurements made on even non-relativistic quantum mechanical systems [17–20,23]. So, we will now study such corrections to a non-relativistic scattering of a scalar particle by a Hermitian spherically symmetric potential $V(R)$. As we will be analyzing a non-relativistic systems, so we will deform the system by a three dimensional spatial length [17–20,23], and not a full four dimensional length in spacetime [24–26]. Even though we will consider only a single scattering process, similar corrections will occur for any scattering process, as these corrections are induced by an internal extended structure in spacetime. Thus, the form of these corrections will be a universal feature of scattering processes, and the main results of this paper can be used to obtain short distance corrections to any scattering processes.

To analyze the effect of the deformation on scattering processes, setting $\hbar = 1$, we will take into account the internal finite distance corrections, that are intrinsic to the scattering process itself. We note that for such simplest non-relativistic scattering on potential $V(R)$ the differential cross-section can be uniquely defined by on-shell scattering amplitude $f^{+}(\mathbf{q};\mathbf{k})$ [29,30], when the initial momentum state $|\mathbf{k}\rangle$ turn to the final momentum state $|\mathbf{q}\rangle$. The scattering amplitudes $f^{\pm}(\mathbf{q};\mathbf{k})$ are coefficients of outgoing or incoming spherical waves as the first order terms of asymptotic expansion of the scattering wave function at the distance $R = |\mathbf{R}| \to \infty$:

$$\Psi^{\pm}_{\mathbf{k}}(\mathbf{R}) \underset{R\to\infty}{\longmapsto} e^{i(\mathbf{k}\cdot\mathbf{R})} + \frac{e^{\pm ikR}}{R} f^{\pm}(\mathbf{q};\mathbf{k}) + O\left(R^{-2}\right), \text{ with} \\ \mathbf{q} = k\mathbf{n}, \;\; \mathbf{k} = k\boldsymbol{\kappa}, \;\; \mathbf{n}^2 = \boldsymbol{\kappa}^2 = 1, \tag{1}$$

where $\mathbf{R} = R\mathbf{n}$ for the spherical coordinates with spherical angles ϑ, ϕ and $\mathbf{n}(\vartheta, \phi) = \mathbf{e}_m \mathrm{n}_m \mapsto (\sin\vartheta\cos\phi, \sin\vartheta\sin\phi, \cos\vartheta)$ in Cartesian basis. The usual elastic differential and total cross-sections are defined by

$$d\sigma = \left|f^{+}(\mathbf{q};\mathbf{k})\right|^2 d\Omega(\mathbf{n}), \text{ and } \sigma = \int \left|f^{+}(k\mathbf{n};k\boldsymbol{\kappa})\right|^2 d\Omega(\mathbf{n}). \tag{2}$$

These cross-sections do not depend on R, and this behavior is important in the quantum scattering theory [29,30]. In order to discuss the full asymptotic expansion of scattering wave function, we note that $\Psi^{\pm}_{\mathbf{k}}(\mathbf{R})$ is a solution to Schrödinger equation for the scattering energy $E > 0$ with $k^2 = 2ME$, potential $U(R) = 2MV(R)$, and the vector operator $\nabla_{\mathbf{R}} = \mathbf{n}\partial_R + R^{-1}\heartsuit_{\mathbf{n}}$ (with the usual angles dependent vector part $\heartsuit_{\mathbf{n}} \mapsto (\partial_\vartheta, (\sin\vartheta)^{-1}\partial_\phi)$ in spherical basis [31]):

$$\left(\nabla^2_{\mathbf{R}} + k^2\right)\Psi^{\pm}_{\mathbf{k}}(\mathbf{R}) = U(R)\Psi^{\pm}_{\mathbf{k}}(\mathbf{R}). \tag{3}$$

It satisfies also the respective Lippmann-Schwinger equation, with $\mathbf{x} = \varrho\mathbf{v}$, $\mathbf{v}^2 = 1$

$$\Psi^{\pm}_{\mathbf{k}}(\mathbf{R}) = e^{i(\mathbf{k}\cdot\mathbf{R})} - \int d^3x \, \frac{e^{\pm ik|\mathbf{R}-\mathbf{x}|}}{4\pi|\mathbf{R}-\mathbf{x}|} U(|\mathbf{x}|)\Psi^{\pm}_{\mathbf{k}}(\mathbf{x}). \tag{4}$$

Now with the operator of angular momentum $\boldsymbol{L}_{\mathbf{n}} = -i(\mathbf{n}\times\heartsuit_{\mathbf{n}})$ and its square $-\heartsuit^2_{\mathbf{n}} = \boldsymbol{L}^2_{\mathbf{n}} \equiv \mathcal{L}_{\mathbf{n}}$, and the self-adjoint operator $\Lambda_{\mathbf{n}} = \sqrt{\mathcal{L}_{\mathbf{n}} + \frac{1}{4}} - \frac{1}{2}$, the Green function admits the following operator form of asymptotic expansion [33,34] for $|\mathbf{x}| = \varrho < R$

$$\frac{e^{\pm ik|\mathbf{R}-\mathbf{x}|}}{4\pi|\mathbf{R}-\mathbf{x}|} = \frac{\chi_{\Lambda_{\mathbf{n}}}(\mp ikR)}{4\pi R} e^{\mp ik(\mathbf{n}\cdot\mathbf{x})} \\ \sim \frac{e^{\pm ikR}}{4\pi R}\left\{1 + \sum_{S=1}^{\infty} \frac{\prod_{\mu=1}^{S}[\mathcal{L}_{\mathbf{n}} - \mu(\mu-1)]}{S!(\mp 2ikR)^S}\right\} e^{\mp ik(\mathbf{n}\cdot\mathbf{x})}. \tag{5}$$

Here the eigenvalue $\mathcal{L}_{\mathbf{n}} \mapsto l(l+1)$, with $\Lambda_{\mathbf{n}} \mapsto l$ for integer l, and the function $\chi_l(y)$ is a sort of well known "spherical" Macdonald function [29–34]). When the potential[1] $U(\varrho)$ has a finite effective range ϱ_0, this expansion allows us to calculate all the pre-asymptotic inverse-power corrections to the wave function $\Psi^{\pm}_{\mathbf{k}}(\mathbf{R})$. Thus, it is possible to write the differential scattering flux $d\Sigma(R)$ as asymptotic series on R^{-S}, similar to (5), at $R \gg \varrho_0$ [33,34],

$$\Psi^{\pm}_{\mathbf{k}}(\mathbf{R}) \underset{R\gg\varrho_0}{\sim} e^{i(\mathbf{k}\cdot\mathbf{R})} + \frac{\chi_{\Lambda_{\mathbf{n}}}(\mp iqR)}{R} f^{\pm}(q\mathbf{n};\mathbf{k}) \\ = e^{i(\mathbf{k}\cdot\mathbf{R})} + \frac{e^{\pm iqR}}{R}\Big\{\sum_{S=0}^{\infty} \frac{h^{\pm}_S(q\mathbf{n};\mathbf{k})}{(\mp 2iqR)^S}\Big\}_{q=k}, \tag{6}$$

[1] It is enough for it to have finite first absolute moment and to decrease at $\varrho \to \infty$ faster than any power of $1/\varrho$ [33].

with $h^{\pm}_S(k\mathbf{n};\mathbf{k}) = \frac{\mathcal{L}_\mathbf{n} - S(S-1)}{S} h^{\pm}_{S-1}(k\mathbf{n};\mathbf{k})$

$$= \frac{1}{S!}\prod_{\mu=1}^{S}[\mathcal{L}_\mathbf{n} - \mu(\mu-1)] f^{\pm}(k\mathbf{n};\mathbf{k}) \tag{7}$$

for amplitude $f^{\pm}(\mathbf{q};\mathbf{k}) = -\frac{1}{4\pi}\int d^3x\, e^{\mp i(\mathbf{q}\cdot\mathbf{x})} U(|\mathbf{x}|)\Psi^{\pm}_\mathbf{k}(\mathbf{x})$,

$$\mathbf{k} = k\boldsymbol{\kappa},\ \ \mathbf{q} = q\mathbf{n},\ \ q \mapsto k, \tag{8}$$

and $\frac{d\Sigma(R)}{d\Omega(\mathbf{n})}$

$$= \frac{1}{2ik}[\overset{*}{f}{}^{+}(q\mathbf{n};k\boldsymbol{\kappa})(\chi_{\overleftarrow{\Lambda}_\mathbf{n}}(iqR)\overleftrightarrow{\partial}_R\chi_{\overrightarrow{\Lambda}_\mathbf{n}}(-iqR))f^{+}(q\mathbf{n};k\boldsymbol{\kappa})]_{q=k}, \tag{9}$$

where $\Sigma(R) \equiv \int \frac{d\Sigma(R)}{d\Omega(\mathbf{n})} d\Omega(\mathbf{n}) = \sigma$, due to

$$(\chi_l(ikR)\overleftrightarrow{\partial}_R\chi_l(-ikR)) = 2ik. \tag{10}$$

The upper arrows indicate the directions of action of the operators, e.g. $\overleftrightarrow{\partial}_R = \overrightarrow{\partial}_R - \overleftarrow{\partial}_R$ and f^* is the complex conjugate of f. The important feature of asymptotic power expansions (5)–(9) is that they still exactly disappear [33] for total (elastic) flux $\Sigma(R)$ (10), which turned to cross-section σ (2) (already not depending on R) due to the self-adjointness of operators $\mathcal{L}_\mathbf{n}$, $\Lambda_\mathbf{n}$ on the unit sphere and the value (10) of Wronskian [29,30]. It may be noted that the influence of the behavior (6) at such finite spatial distance R onto event rate seems important for explanation of reactor anomaly in neutrino flavor oscillations [34–36].

We will now analyze a short distance correction to such a system due to the existence of a minimal measurable length in spacetime. This will be done by analyzing the effects of a minimal measurable length on the asymptotic expansion of scattering wave function (6) and then on the differential scattering flux (9). Now the scattering wave function $\Psi^{\pm}_{\mathbf{l}_j}(\mathbf{R})$ being a positive energy ($E > 0$) solution of Schrödinger equation would satisfy the Lippmann-Schwinger equation deformed by the existence of a minimal length in spacetime. The existence of a minimal measurable length scale in turn deforms the usual uncertainty principle from $\Delta x\Delta p \geqslant 1/2$ to a generalized uncertainty principle: $\Delta\widehat{x}_l\Delta\widehat{p}_l \geqslant |(1+3\beta\langle\langle\widehat{p}_l^2\rangle\rangle)|/2$, and $\Delta\widehat{x}_l\Delta\widehat{p}_m \geqslant |\beta\langle\langle\widehat{p}_l\widehat{p}_m\rangle\rangle| \geqslant 0$ for $l \neq m$, where β is a small perturbative dimension parameter of the deformation [4,37,38]. Since $\langle\langle\widehat{\mathbf{p}}^2\rangle\rangle \geqslant \langle\langle\widehat{p}_l^2\rangle\rangle$ holds for averaging over any quantum state, the generalized uncertainty principle in turn deforms the Heisenberg algebra, and deformed operators satisfy [38–42]:

$$[\widehat{x}_l, \widehat{p}_m] = i\big[\delta_{lm} + \beta\big(\delta_{lm}\widehat{\mathbf{p}}^2 + 2\widehat{p}_l\widehat{p}_m\big)\big]$$
$$\approx i\big[\delta_{lm} + \beta\big(\delta_{lm}\mathbf{p}^2 + 2p_lp_m\big)\big],$$
$$l, m = 1, 2, 3,\quad \widehat{\mathbf{p}}^2 = \widehat{p}_l\widehat{p}_m\delta_{lm}. \tag{11}$$

This deformed Heisenberg algebra of deformed operators $\widehat{x}_l$, $\widehat{p}_m$ can be related perturbatively for small β to the usual Heisenberg algebra $[x_l, p_m] = i\delta_{lm}$, with usual representation of $\mathbf{p} = -i\nabla_\mathbf{x}$ for $p_lp_m\delta_{lm} = \mathbf{p}^2$, as $\widehat{x}_m = x_m$ and $\widehat{\mathbf{p}} = \mathbf{p}(1+\beta\mathbf{p}^2) = -i\nabla_\mathbf{x}(1-\beta\nabla^2_\mathbf{x})$ [38–42]. It may be noted that other sources of such deformations of the Heisenberg algebra have been motivated by non-locality [43], doubly special relativity [44,45], deformed dispersion relations in the bosonic string theory [46], Horava-Lifshitz gravity [47,48], discrete spacetime [49], models based on string field theory [50], spacetime foam [51], spin-network [52], and noncommutative geometry [53]. So, we will deform the coordinate representation of the momentum operator in such a general way, that the free Hamiltonian (for $\mathbf{x} \mapsto \mathbf{R}$) is deformed as

$$H_0 = \mathbf{p}^2 = -\nabla^2_\mathbf{R} \longmapsto \widetilde{H}_0 = (\widehat{\mathbf{p}})^2 = \mathbf{p}^2 + \beta g\big(\mathbf{p}^2\big)$$
$$= -\nabla^2_\mathbf{R} + \beta g\big(-\nabla^2_\mathbf{R}\big), \tag{12}$$

where $g(-\nabla^2_\mathbf{R})$ is a real differentiable function of the $-\nabla^2_\mathbf{R}$, such that $g(0) = 0 = g'(0)$. Strictly speaking, it describes only first term of low energy expansion of some more general Hamiltonian.[2] Now the deformation (11) of the Heisenberg algebra, produces a polynomial of order $g(-\nabla^2_\mathbf{R}) \mapsto 2(-\nabla^2_\mathbf{R})^N$ with $N = 2$ in the Hamiltonian (12). However, as shown below, the results obtained here can be easily generalized to any general polynomial function $g(z)$ of $z = -\nabla^2_\mathbf{R}$, containing any powers z^n with $2 \leqslant n \leqslant N$ (for arbitrary $N > 2$). The dimension of β is always determined by the lowest value of n.

The deformation of the Lippmann-Schwinger equation can be obtained from the deformation of stationary Schrödinger equation (3). Now in coordinate representation of momentum operator the deformation of the above free Hamiltonian (12), can be written as

$$\big[-\widetilde{H}_0 + k^2\big]\Psi^{\pm}_{\mathbf{l}_j}(\mathbf{R}) = U(R)\Psi^{\pm}_{\mathbf{l}_j}(\mathbf{R}). \tag{13}$$

Thus, the wave function for this system satisfies the deformed Lippmann-Schwinger equation

$$\Psi^{\pm}_{\mathbf{l}_j}(\mathbf{R}) = e^{i(\mathbf{l}_j\cdot\mathbf{R})} - \int d^3x\, \mathcal{G}^{(\pm)}_k(\mathbf{R}-\mathbf{x})\, U(|\mathbf{x}|)\Psi^{\pm}_{\mathbf{l}_j}(\mathbf{x}). \tag{14}$$

It depends on solutions of free problem with the assumed entire (polynomial) function $g(z)$ as

$$\widetilde{H}_0\, e^{i(\mathbf{l}_j\cdot\mathbf{R})} = k^2 e^{i(\mathbf{l}_j\cdot\mathbf{R})} \equiv \mathcal{E}\, e^{i(\mathbf{l}_j\cdot\mathbf{R})}, \qquad \mathbf{l}_j = \ell_j(k)\boldsymbol{\kappa}, \tag{15}$$

$$\big[\nabla^2_\mathbf{R} - \beta g\big(-\nabla^2_\mathbf{R}\big) + k^2\big]\mathcal{G}^{(\pm)}_k(\mathbf{R}-\mathbf{x}) = -\delta_3(\mathbf{R}-\mathbf{x}), \tag{16}$$

$$\mathcal{G}^{(\pm)}_k(\mathbf{R}-\mathbf{x}) \equiv \int \frac{d^3q}{(2\pi)^3}\, e^{i(\mathbf{q}\cdot(\mathbf{R}-\mathbf{x}))}\, F(\mathbf{q}). \tag{17}$$

Since Eq. (16) leads to the equation $[\mathbf{q}^2 + \beta g(\mathbf{q}^2) - k^2]F(\mathbf{q}) = 1$, the above relation can be expressed as

$$\mathcal{G}^{(\pm)}_k(\mathbf{R}-\mathbf{x}) = \int \frac{d^3q}{(2\pi)^3}\frac{e^{i(\mathbf{q}\cdot(\mathbf{R}-\mathbf{x}))}}{[\mathbf{q}^2 + \beta g(\mathbf{q}^2) - k^2 \mp i0]},\ \text{where}$$
$$|\mathbf{R}-\mathbf{x}| = r. \tag{18}$$

For $\Phi(q^2) = q^2 + \beta g(q^2) - k^2$, with $\mathbf{q} = q\mathbf{n}'$ and $d^3q = q^2dqd\Omega(\mathbf{n}')$, the Green function is reduced to

$$\mathcal{G}^{(+)}_k(\mathbf{R}-\mathbf{x}) = \frac{2}{(2\pi)^2r}\int_0^\infty \frac{dq\, q\, \sin qr}{\Phi(q^2) - i0}$$
$$= \frac{1}{i(2\pi)^2r}\int_{-\infty}^{\infty}\frac{dq\, q\, e^{iqr}}{\Phi(q^2) - i0} =$$
$$= \frac{2\pi i}{i(2\pi)^2r}\sum_{s=1}^{N}\text{res}\left(\frac{q\, e^{iqr}}{\Phi(q^2)}\right)|_{q=\ell_s}$$
$$= \frac{1}{4\pi r}\sum_{s=1}^{N}\frac{e^{i\ell_s r}}{\Phi'(\ell_s^2)}, \tag{19}$$

[2] Similar to first relativistic correction from the expansion of $E_p = c\sqrt{\mathbf{p}^2 + (Mc)^2} - Mc^2 \approx \mathbf{p}^2/(2M) - (\mathbf{p}^2)^2/(8M^3c^2)$. The respective Lagrangian picture for the case (12) corresponding to (11) is discussed below.

$$\text{where } \Phi\big(q^2\big)=0, \text{ for } q^2=\ell_s^2, \tag{20}$$
$$\text{with } \Phi'\big(\ell_s^2\big)=1+\beta g'\big(\ell_s^2\big)\neq 0, \text{ and}$$
$$Im\,\ell_s>0, \text{ or } \ell_s\mapsto \ell_s+i0, \text{ for } Im\,\ell_s=0. \tag{21}$$

We only need to consider the first-order poles, because the poles of higher orders will produce positive powers of r, destroying the asymptotic expansion (5). The real polynomial function $\Phi(q^2)$ has only real or complex conjugate zeros ℓ_s^2 (20), which satisfy the properties given in (21). For example, when all the contributions to (19) (for $s\geqslant 2$) have $Im\,\ell_s>0$ and vanish exponentially with $r\to\infty$, the only one zero (20) $\ell_1=\ell_1(k)>0$ of function $\Phi(q^2)$, admits the radiation condition of (21). So in expansion (19) for $\mathbf{x}=\varrho\mathbf{v}$, $r=|\mathbf{R}-\mathbf{x}|\gg 1/Im\,\ell_s$ with $s\geqslant 2$, we need to keep only this zero, substituting (in the sense of asymptotic expansion)

$$\mathcal{G}_k^{(+)}(\mathbf{R}-\mathbf{x})\longmapsto \frac{e^{i\ell_1|\mathbf{R}-\mathbf{x}|}}{4\pi\,\Phi'(\ell_1^2)|\mathbf{R}-\mathbf{x}|} \tag{22}$$

into Eq. (14). Instead of amplitude (8), it leads to the on-shell scattering amplitude, containing the corrections depending on β due to $\ell_1(k)$ and $\Phi'(\ell_1^2)$ as

$$f_{11}^{+}(\mathbf{q};\mathbf{l}_1)=-\frac{1}{4\pi\,\Phi'(\ell_1^2)}\int d^3x\, e^{-i(\mathbf{q}\cdot\mathbf{x})}U(|\mathbf{x}|)\Psi_{\mathbf{l}_1}^{+}(\mathbf{x}). \tag{23}$$

So, in this situation one can uniquely define the scattering wave function, scattering amplitude and differential scattering flux by simple substitutions of $\mathbf{k}\mapsto\mathbf{l}_1=\ell_1\boldsymbol{\kappa}$, i.e., $k\mapsto\ell_1=\ell_1(k)$, $\mathbf{q}=q\mathbf{n}$, $q\mapsto\ell_1$, everywhere in Eqs. (5) – (10), with the respective redefinitions of amplitude (23) and incoming flux. We are only interested in the physical wave function, which defines the physical amplitude (and differential scattering flux (9))

$$\Psi_{\mathbf{l}_1}^{+}(\mathbf{R}) \underset{R\gg\varrho_0}{\sim} e^{i(\mathbf{l}_1\cdot\mathbf{R})}+\frac{\chi_{\Lambda_\mathbf{n}}(-iqR)}{R}\,f_{11}^{+}(\mathbf{q};\mathbf{l}_1)$$
$$\underset{R\to\infty}{\longmapsto} e^{i(\mathbf{l}_1\cdot\mathbf{R})}+\frac{e^{i\ell_1 R}}{R}\,f_{11}^{+}(\mathbf{q};\mathbf{l}_1)+O\big(R^{-2}\big). \tag{24}$$

Indeed, there always exists the single perturbative zero of $\Phi(q^2)$ as a solution of Eq. (20), which for small $\beta\to 0$ goes to k^2 as $\ell_s^2=k^2-\beta g(\ell_s^2)\mapsto\ell_1^2\approx k^2-\beta g(k^2)$, while other solutions turn to infinity[3] like $\beta^{-2\epsilon}$ with $\epsilon>0$. For example, for the case (11), (12) with $g(z)=2z^2$, $N=2$, $\epsilon=1/2$ and $\xi=\beta k^2$, they are defined with $\ell_s(k)=[\ell_s^2(k)]^{1/2}$ for $s=1,2$, as

$$\left.\begin{matrix}\ell_1^2(k)\\ \ell_2^2(k)\end{matrix}\right\}=\frac{-1\pm\sqrt{1+8\xi}}{4\beta}=\frac{2k^2}{1\pm\sqrt{1+8\xi}}$$
$$=\begin{cases}\ell_1^2(k)=k^2(1-2\xi+8\xi^2+\ldots), & \text{for } |\xi|\ll 1,\\ \ell_2^2(k)=-\dfrac{1}{2\beta}-\ell_1^2(k)=-\dfrac{k^2}{2\beta\ell_1^2(k)}.\end{cases} \tag{25}$$

So, for $\beta>0$, $\xi>0$, $\forall k^2>0$ with the main branches of all square roots, such as $[-\ell_s^2\mp i0]^{1/2}=\mp i\ell_s+0$ (when $\ell_s>0$), we have the above situation with $\ell_1(k)>0$, $\ell_2(k)=i|\ell_2(k)|$. For $\beta<0$, $\xi<0$, we have $\ell_2(k)>\ell_1(k)>0$ only until $8|\xi|<1$. For $8|\xi|>1$, the roots $\ell_{1,2}^2(k)$ are complex conjugate, with $Im\,\ell_{1,2}>0$ and there are no scattering waves. In general case for $N>2$, the Green function (19) is also a multivalued function of scattering energy $\mathcal{E}=k^2$ that admits an extraction of a single-valued branch in the cutted $\mathcal{E}$-plane, with the cuts for every square root $[-\ell_s^2(k)]^{1/2}$ of the wave numbers (25). The first physical sheet of its Riemann surface is defined by physical cut [29], that goes along $\mathcal{E}\geqslant 0$, arising due to square root of perturbative wave number $[-\ell_1^2(k)]^{1/2}$. Then all other cuts with $s\geqslant 2$ are on the next sheets. For the above case (11), (12), with $N=2$, $\epsilon=1/2$, the next "kinematical" cut from the point $\mathcal{E}=-(8\beta)^{-1}$ (for small β) lies far away from the origin on the first sheet (for $\beta>0$). Besides, it is screened by physical cut for $\beta<0$.

[3] That gives an essentially singular point at $\beta=0$ for the function $e^{i\ell_s(k)R}$. We call such $\ell_s(k)$ as non-perturbative ones.

Actually, the free states of scattering theory may be determined by any real branch (21) of spectrum of free Hamiltonian (15). So, we are dealing with a sort of multichannel problem [29]. The point is that every s-term in the sum (19) with ℓ_s from Eq. (20), being a solution to free Eq. (3) (with $U=0$) for $k\mapsto\ell_s$, is a solution to homogeneous Eq. (16) for $r>0$, but only the full sum (19) gives the solution (18) to non-homogeneous Eq. (16) for $r\geqslant 0$. After substitution of the relations (19), (5) into Eq. (14), the asymptotic expansion of wave function for arbitrary real j-mode $\ell_j=\ell_j(k)$, with $1\leqslant j,s\leqslant\overline{N}\leqslant N$ and $\mathbf{R}=R\mathbf{n}$, $\mathbf{l}_j=\ell_j\boldsymbol{\kappa}$, $\mathbf{q}_s=q_s\mathbf{n}$, $q_s=\ell_s$, can be written as

$$\Psi_{\mathbf{l}_j}^{+}(\mathbf{R}) \underset{R\gg\varrho_0}{\sim} e^{i(\mathbf{l}_j\cdot\mathbf{R})}+\sum_{s=1}^{\overline{N}}\frac{\chi_{\Lambda_\mathbf{n}}(-iq_sR)}{R}\,f_{sj}^{+}(\mathbf{q}_s;\mathbf{l}_j)$$
$$\underset{R\to\infty}{\longmapsto} e^{i(\mathbf{l}_j\cdot\mathbf{R})}+\sum_{s=1}^{\overline{N}}\frac{e^{iq_sR}}{R}\,f_{sj}^{+}(\mathbf{q}_s;\mathbf{l}_j), \tag{26}$$

with the scattering amplitudes:

$$f_{sj}^{+}(\mathbf{q}_s;\mathbf{l}_j)=-\frac{1}{4\pi\,\Phi'(q_s^2)}\int d^3x\, e^{-i(\mathbf{q}_s\cdot\mathbf{x})}U(|\mathbf{x}|)\Psi_{\mathbf{l}_j}^{+}(\mathbf{x}). \tag{27}$$

Now they look like a multichannel amplitudes [29], between different channels (modes), if $q_s\neq\ell_j$, i.e. $s\neq j$ (where any real j-mode scatters into all other possible real s-modes).

Let us firstly neglect (for small β) the difference between the exactly conserved current and the usually defined diagonal current [29–31,33], and assume that (marking this assumption as (!))

$$R^2d\Omega(\mathbf{n})(\mathbf{n}\cdot\mathbf{J}_{\mathbf{l}_j,\mathbf{l}_j}[\Psi(\mathbf{R})])\overset{(!)}{\longmapsto}R^2d\Omega(\mathbf{n})\frac{1}{2i}(\overset{*}{\Psi}{}_{\mathbf{l}_j}^{+}(\mathbf{R})\overset{\leftrightarrow}{\partial}_R\Psi_{\mathbf{l}_j}^{+}(\mathbf{R})),$$
$$\text{with } \overset{\leftrightarrow}{\partial}_R=(\mathbf{n}\cdot\overset{\leftrightarrow}{\nabla}_R). \tag{28}$$

Now the sums (26) at bilinear form of $\Psi_{\mathbf{l}_j}^{+}(\mathbf{R})$ in (28) lead to the sum of R-dependent interference terms proportional to $e^{i(\ell_s-\ell_\nu)R}$. However, the usually assumed macroscopic averaging over R over detector volume [29] recasts these rapidly oscillating exponentials as $\langle\langle e^{i(\ell_s-\ell_\nu)R}\rangle\rangle=\delta_{s\nu}$. Thus, repeating all the steps of [33], we come to the form of differential scattering flux as diagonal sum of the inclusive expressions (similar to (9)) with $k\mapsto\ell_j$, $q\mapsto q_s=\ell_s$. Now using (5)–(7) in (26), up to three leading orders in $(\ell_sR)^{-1}$, and under the assumption (!) in (28), we can write

$$\frac{d\Sigma_j(R)}{d\Omega(\mathbf{n})}\overset{(!)}{\longmapsto}\sum_{s=1}^{\overline{N}}\frac{q_s}{\ell_j}\Big\{|f_{sj}^{+}(q_s\mathbf{n};\,\ell_j\boldsymbol{\kappa})|^2$$
$$-\frac{1}{(q_sR)}\,Im\big(\overset{*}{f}{}_{sj}^{+}(q_s\mathbf{n};\,\ell_j\boldsymbol{\kappa})\mathcal{L}_\mathbf{n}f_{sj}^{+}(q_s\mathbf{n};\,\ell_j\boldsymbol{\kappa})\big)+$$
$$+\frac{1}{4(q_sR)^2}\big[|\mathcal{L}_\mathbf{n}f_{sj}^{+}(q_s\mathbf{n};\,\ell_j\boldsymbol{\kappa})|^2$$
$$-Re\big(\overset{*}{f}{}_{sj}^{+}(q_s\mathbf{n};\,\ell_j\boldsymbol{\kappa})\mathcal{L}_\mathbf{n}^2f_{sj}^{+}(q_s\mathbf{n};\,\ell_j\boldsymbol{\kappa})\big)\big]+O\left(\frac{1}{(q_sR)^3}\right)\Big\}. \tag{29}$$

In accordance with (10), every correction here disappears separately under integration over solid angle due to the self adjointness of operator $\mathcal{L}_{\mathbf{n}}$ on the unit sphere. For $g(z)=2z^N$ one has $1/\epsilon = 2(N-1)$. For very small dimensionless deformation parameter $\xi=\beta k^{2(N-1)} \ll 1$, every real non-perturbative wave number $\ell_s>0$, $s\geqslant 2$ in sum (26) turns to infinity like $\ell_s \sim |\beta|^{-\epsilon}$ (similarly, $\ell_2(k)$ (25) for $\beta<0$). At the first sight, this leads to extremely rapid oscillations of $\exp\{i\ell_s R\}$, that can be neglected in (26), leaving us again only with the unique scattering solution (23), (24).

However, this is not the case. Up to the values of scattering amplitudes, the contributions of these wave numbers in asymptotic expansion (9), (29) are suppressed again only by the inverse powers of $\ell_s R$. Moreover, since the Born approximation becomes relevant for the wave functions (26) $\Psi^{+}_{\mathbf{l}_j}(\mathbf{R}) \approx e^{i(\mathbf{l}_j\cdot\mathbf{R})}$ for $j\geqslant 2$ as well as for the scattering amplitude (27), the last arises as real Fourier image of a real function $U(\varrho)$, and gives zero contributions to the second term in (29) of order $(\ell_s R)^{-1}$ (as well as of order $(\ell_s R)^{-3}$). For the potentials $U(\varrho)$, non-singular at $\varrho\to 0$ [29,30], the amplitudes disappear fast enough with the square of momentum transfer $\mathbf{Q}^2_{sj}=(\mathbf{q}_s-\mathbf{l}_j)^2\to\infty$:

$$f^{+}_{sj}(\mathbf{q}_s;\mathbf{l}_j)\approx f^{+B}_{sj}(\mathbf{q}_s;\mathbf{l}_j) = -\frac{1}{4\pi\,\Phi'(q_s^2)}\int d^3\mathrm{x}\, e^{-i(\mathbf{Q}_{sj}\cdot\mathbf{x})}U(|\mathbf{x}|),$$
$$\mathbf{Q}_{sj}=\mathbf{q}_s-\mathbf{l}_j,\ \text{ e.g. for}$$
$$U(\varrho)=\alpha\varrho^{2\eta-2},\ \ \eta>0,\ \text{ that is}$$
$$f^{+B}_{sj}(\mathbf{q}_s;\mathbf{l}_j) = -\alpha\sin(\pi\eta)\Gamma(2\eta)|\mathbf{Q}_{sj}|^{-1-2\eta}\big[\Phi'\big(q_s^2\big)\big]^{-1}, \tag{30}$$

where for $s\neq j\geqslant 2$ one has $|\mathbf{Q}_{sj}|\sim\ell_j\sim|\beta|^{-\epsilon}\to\infty$. Since $\mathbf{Q}^2_{sj}=\ell_s^2+\ell_j^2-2\ell_s\ell_j\cos\vartheta$ for $\boldsymbol{\kappa}=\mathbf{e}_3$, then the operator $\mathcal{L}_{\mathbf{n}}$, being for the spherically symmetric case (27) the second order differential operator with respect to the $\cos\vartheta$ only [29], will be the similar operator with respect to the $\mathbf{Q}^2_{sj}$ for every term in (29).

To make a self consistent calculations one should take into account the change of conserved current supplementing the change in the free Hamiltonian (12). Now we consider the corrections to the currents, for the case with $g(z)=2z^2$, due to difference of exact conserved current from the above assumed simple one (!) (28). For the general field theory, with the higher (second) derivatives the Lagrangian of complex classical scalar field depends on its variables as $\mathcal{F}=\mathcal{F}(\psi,\psi^*;\partial_\mu\psi,\partial_\mu\psi^*;\partial_\lambda\partial_\gamma\psi,\partial_\lambda\partial_\gamma\psi^*)$. Now for the field variation $\delta\psi$, we can write $\delta(\partial_\mu\psi)=\partial_\mu(\delta\psi)$. So, the variation of the action $\delta\mathcal{I}[\psi,\psi^*]=\delta_\psi\mathcal{I}+\delta_{\psi^*}\mathcal{I}$ (for the action $\mathcal{I}[\psi,\psi^*]=\int d^4x\mathcal{F}(\psi,\psi^*;\ldots)$, and using $\partial_\mu=(\partial_0,\nabla_{\mathbf{x}})$), can be expressed as [31,54]

$$\begin{aligned}\delta\mathcal{I}\big[\psi,\psi^*\big] &= \int d^4x \sum_{\varphi=\psi^*,\psi}\delta\varphi\Big[\frac{\delta\mathcal{F}}{\delta\varphi}-\partial_\mu\Big(\frac{\delta\mathcal{F}}{\delta(\partial_\mu\varphi)}\Big)\\ &+\partial_\lambda\partial_\gamma\Big(\frac{\delta\mathcal{F}}{\delta(\partial_\lambda\partial_\gamma\varphi)}\Big)\Big]+\\ &+\int d^4x\,\partial_\mu\sum_{\varphi=\psi^*,\psi}\Big\{\delta\varphi\Big[\frac{\delta\mathcal{F}}{\delta(\partial_\mu\varphi)}-\partial_\gamma\Big(\frac{\delta\mathcal{F}}{\delta(\partial_\gamma\partial_\mu\varphi)}\Big)\Big]\\ &+\frac{\delta\mathcal{F}}{\delta(\partial_\mu\partial_\gamma\varphi)}\,\partial_\gamma(\delta\varphi)\Big\}.\end{aligned} \tag{31}$$

The vanishing of the expressions in square brackets in first integral in (31), represent the equations of motion. The vanishing of the expression for four-divergence in second integral in (31)), defines the respective conserved current. Now the equation of motion for the Schrödinger fields $i\partial_0\psi=(\widetilde{H}_0+U(\mathbf{x}))\psi$ [31], corresponding to Schrödinger Eq. (13), with the Hamiltonian (12), for $g(z)=2z^2$, is produced by the first square brackets of Eq. (31), with the following non-relativistic Lagrangian, for suitable chosen units and with $n,m=1,2,3$

$$\begin{aligned}\mathcal{F} &= i\big(\psi^*\partial_0\psi-\psi\partial_0\psi^*\big)-\big(\big(\nabla\psi^*\big)\cdot(\nabla\psi)\big)-U(\mathbf{x})\psi^*\psi\\ &-2\beta\big(\nabla_n\nabla_m\psi^*\big)(\nabla_n\nabla_m\psi).\end{aligned} \tag{32}$$

So, for the global gauge transformation [31], $\delta\psi=i\psi\delta\alpha$, $\delta\psi^*=-i\psi^*\delta\alpha$, $\partial_\mu\delta\alpha=0$, the second integral in Eq. (31) define the gauge current $J^\mu=(J^0,\mathbf{J})$ with $J^0[\psi]=\psi^*\psi$ and

$$\begin{aligned}2i\mathbf{J}[\psi] &= \big[\psi^*\nabla_{\mathbf{R}}\psi-2\beta\,\psi^*\nabla_{\mathbf{R}}\big(\nabla^2_{\mathbf{R}}\psi\big)\\ &+2\beta\,\big(\nabla_m\psi^*\big)\nabla_{\mathbf{R}}(\nabla_m\psi)\big]-[\ldots]^*,\ \text{ that is}\end{aligned} \tag{33}$$

$$\begin{aligned}2i\mathbf{J}[\psi] &= \big[\psi^*\nabla_{\mathbf{R}}\psi-4\beta\,\psi^*\nabla_{\mathbf{R}}\big(\nabla^2_{\mathbf{R}}\psi\big)\\ &+2\beta\,\nabla_m\big(\psi^*\nabla_{\mathbf{R}}(\nabla_m\psi)\big)\big]-[\ldots]^*.\end{aligned} \tag{34}$$

This is exactly conserved even for non-diagonal case because $(\nabla\cdot\mathbf{J}_{\mathbf{l}_j\mathbf{l}_s}[\psi])=0$ for any stationary scattering solutions $\psi=\Psi^{\pm}_{\mathbf{l}_j}(\mathbf{R})$ to Schrödinger Eq. (13). It is easy to see that the full three-divergence in the third term in Eq. (34) does not give any contribution to the incoming flux, $\mathbf{J}[e^{i(\mathbf{l}_j\cdot\mathbf{R})}]=\mathbf{l}_j(1+4\beta\ell_j^2)$. Moreover, at least up to the order of $(\ell_s R)^{-4}$, this third term does not give any contribution to the differential scattered flux (29) generated by radial scattered flux Eq. (28). This is because for the current (34), there is $(\mathbf{n}\cdot\mathbf{J}[e^{iq_sR}/R])=q_s(1+4\beta q_s^2)$, and every summand of the sum over s in the first pre-asymptotic relation (26) (and every s-term in the sum (19)), satisfies [33] free equation (3) as $(\nabla^2_{\mathbf{R}}+q_s^2)\psi_{q_s}(\mathbf{R})=0$ for $\mathbf{R}\neq 0$. Eventually, for the case with $g(z)=2z^2$, up to the order of $(\ell_s R)^{-4}$, these currents corrections only renormalize the external multipliers of differential scattering flux (29) as

$$\frac{q_s}{\ell_j}\longmapsto\frac{q_s}{\ell_j}\frac{(1+4\beta q_s^2)}{(1+4\beta\ell_j^2)}=\frac{q_s}{\ell_j}\frac{\Phi'(q_s^2)}{\Phi'(\ell_j^2)}\equiv\frac{q_s}{\ell_j}\frac{\widetilde{H}'_0(q_s^2)}{\widetilde{H}'_0(\ell_j^2)}=\frac{v_s}{v_j},\ \text{ where}$$
$$v_j=\frac{\partial\widetilde{H}_0(\ell_j^2)}{\partial\ell_j}. \tag{35}$$

Here v_j is the velocity. This result looks quite general and conforms to the physical meaning of currents, differential scattering flux and cross-section. Now with this substitution (denoted below by superscript ^{ren}) we can use the expression (29) without the assumption (!) of (28). Thus, we can use at least two additional terms of orders of $(\ell_s R)^{-3}$ and $(\ell_s R)^{-4}$. They are obtained from the general expression (5)–(9), with the following replacement in (29)

$$\begin{aligned}&O\Big(\frac{1}{(q_sR)^3}\Big)\\ &\longmapsto\frac{1}{3(2q_sR)^3}Im\big[f^*_{sj}\mathcal{L}^3_{\mathbf{n}}f_{sj}-3(\mathcal{L}_{\mathbf{n}}f_{sj})^*\mathcal{L}^2_{\mathbf{n}}f_{sj}-2f^*_{sj}\mathcal{L}^2_{\mathbf{n}}f_{sj}\big]+\\ &+\frac{1}{12(2q_sR)^4}\big\{3|\mathcal{L}^2_{\mathbf{n}}f_{sj}|^2+Re\big[f^*_{sj}\mathcal{L}^4_{\mathbf{n}}f_{sj}-4(\mathcal{L}_{\mathbf{n}}f_{sj})^*\mathcal{L}^3_{\mathbf{n}}f_{sj}\big]\\ &+12\big[Re\big(f^*_{sj}\mathcal{L}^2_{\mathbf{n}}f_{sj}\big)-|\mathcal{L}_{\mathbf{n}}f_{sj}|^2\big]-\\ &-8Re\big[f^*_{sj}\mathcal{L}^3_{\mathbf{n}}f_{sj}-(\mathcal{L}_{\mathbf{n}}f_{sj})^*\mathcal{L}^2_{\mathbf{n}}f_{sj}\big]\big\},\ \text{ where for short:}\end{aligned}$$
$$f_{sj}=f^{+}_{sj}(q_s\mathbf{n};\ell_j\boldsymbol{\kappa}),\ \ (\mathcal{L}_{\mathbf{n}}f_{sj})^*=f^*_{sj}\overleftarrow{\mathcal{L}}_{\mathbf{n}}. \tag{36}$$

We see that for non-perturbative modes $\ell_j(k)$ with $j\geqslant 2$, we only have the real Born amplitudes of type (30). They can contribute only to the even powers of R^{-S} with $S=0,2,4,\ldots$, in expansion of differential scattering fluxes. The respective expressions (29), (36), renormalized by the replacement (35), are the main result of this work. Rewriting them with obvious notations of the summands $d\Sigma^{ren}_{sj}(R)/d\Omega(\mathbf{n})$ as

$$\frac{d\Sigma^{ren}_{j}(R)}{d\Omega(\mathbf{n})}=\sum_{s=1}^{\overline{N}}\frac{d\Sigma^{ren}_{sj}(R)}{d\Omega(\mathbf{n})}, \quad \text{we define also}$$

$$\overline{\frac{d\Sigma^{ren}(R)}{d\Omega(\mathbf{n})}}=\sum_{j=1}^{\overline{N}}\frac{d\Sigma^{ren}_{j}(R)}{d\Omega(\mathbf{n})}. \tag{37}$$

It should be stressed that these quantities actually are those that measured experimentally at a finite distance R as differential cross-sections. Their intrinsic R-dependence given here is defined only by observable quantities like the on-shell scattering amplitudes (23), (27) or partial phase shifts [33]. So, this intrinsic R-dependence seems to be sensitive to the corrections from existence of the minimal measurable length and can provide an additional opportunity for experimental resolution of these corrections. When the experimental resolution will permit to distinguish between these different renormalized differential scattering fluxes (37) at different R, the perturbative mode $j=1$ with different $s\geqslant 1$ modes (arised in Eqs. (26, (27), (29), (36)) will be the most interesting for observation due to the amplification factor (35). Now for the total cross-sections all the inverse-power terms in (29), (36) disappear again, and similar to (10), we obtain

$$\Sigma^{ren}_{sj}(R)=\frac{v_s}{v_j}\int\left|f^{+}_{sj}(\ell_s\mathbf{n};\ell_j\boldsymbol{\kappa})\right|^2 d\Omega(\mathbf{n})=\sigma^{ren}_{sj}, \quad \text{and}$$

$$\sigma^{ren}_{j}=\sum_{s=1}^{\overline{N}}\sigma^{ren}_{sj}, \qquad \overline{\sigma}^{ren}=\sum_{j=1}^{\overline{N}}\sigma^{ren}_{j}. \tag{38}$$

So, such a measurement would be interesting when the experimental resolution at least between σ^{ren}_{11}, σ^{ren}_{1}, and $\overline{\sigma}^{ren}$ would be achieved.

In this paper, we have analyzed the short distance corrections to scattering processes. These corrections occur due to the existence of a minimal measurable length scale in spacetime. The existence of a minimal measurable length scale deforms the Heisenberg algebra, which in its turn deforms the coordinate representation of the momentum operator. The deformation of the momentum operator produces the higher derivative corrections to the free Hamiltonian and, besides the changes of different physical processes, modifies the Lippmann-Schwinger equation. The modification of the Lippmann-Schwinger equation modifies the description of scattering processes. We explicitly calculate corresponding corrections to the Green function and to conserved current for these processes. The obtained modification of scattering amplitudes determine the corrections to the observable cross-sections and to the R- dependent differential scattering fluxes defined recently in [33]. So, it is justified that the existence of a minimal measurable length regardless to its origin can correct scattering processes, and scattering experiments with finite macroscopically small base R can in principle detect such corrections.

It may be noted that the results obtained in this paper are quite general and can be applied to most non relativistic scattering processes, where they would act as universal corrections to all scattering processes due to an extended structure in spacetime. Such corrections would be observed at intermediate scale. It will be interesting to use these results to analyze specific scattering processes, and to obtain new bounds for the existence of a minimal measurable length scale in spacetime.

Acknowledgements

Authors thank M.V. Polyakov and A.E. Rastegin for useful discussions and Referee for constructive comments and valuable advice.

References

[1] D. Amati, M. Ciafaloni, G. Veneziano, Phys. Lett. B 216 (1989) 41.
[2] A. Kempf, G. Mangano, R.B. Mann, Phys. Rev. D 52 (1995) 1108.
[3] L.N. Chang, D. Minic, N. Okamura, T. Takeuchi, Phys. Rev. D 65 (2002) 125028.
[4] S. Benczik, L.N. Chang, D. Minic, N. Okamura, S. Rayyan, T. Takeuchi, Phys. Rev. D 66 (2002) 026003.
[5] S. Hossenfelder, Living Rev. Relativ. 16 (2013) 2.
[6] M.R. Douglas, D.N. Kabat, P. Pouliot, S.H. Shenker, Nucl. Phys. B 485 (1997) 85.
[7] A. Smailagic, E. Spallucci, T. Padmanabhan, arXiv:hep-th/0308122.
[8] M. Fontanini, E. Spallucci, T. Padmanabhan, Phys. Lett. B 633 (2006) 627.
[9] C. Hull, B. Zwiebach, J. High Energy Phys. 0909 (2009) 099.
[10] V.E. Marotta, F. Pezzella, P. Vitale, J. High Energy Phys. 1808 (2018) 185.
[11] M. Faizal, A.F. Ali, S. Das, Int. J. Mod. Phys. A 32 (2017) 1750049.
[12] M. Maggiore, Phys. Lett. B 304 (1993) 65.
[13] M.I. Park, Phys. Lett. B 659 (2008) 698.
[14] P. Dzierzak, J. Jezierski, P. Malkiewicz, W. Piechocki, Acta Phys. Pol. B 41 (2010) 717.
[15] R. Percacci, G.P. Vacca, Class. Quantum Gravity 27 (2010) 245026.
[16] T. Padmanabhan, Class. Quantum Gravity 4 (1987) L107.
[17] S. Das, E.C. Vagenas, Phys. Rev. Lett. 101 (2008) 221301.
[18] S. Das, E.C. Vagenas, Phys. Rev. Lett. 104 (2010) 119002.
[19] A.F. Ali, S. Das, E.C. Vagenas, Phys. Rev. D 84 (2011) 044013.
[20] I. Pikovski, M.R. Vanner, M. Aspelmeyer, M. Kim, C. Brukner, Nat. Phys. 8 (2012) 393.
[21] V.V. Nesvizhevsky, et al., Nature 415 (2002) 297.
[22] V.V. Nesvizhevsky, et al., Phys. Rev. D 67 (2003) 102002.
[23] P. Pedram, K. Nozari, S.H. Taheri, J. High Energy Phys. 1103 (2011) 093.
[24] M. Kober, Int. J. Mod. Phys. A 26 (2011) 4251.
[25] M. Faizal, S.I. Kruglov, Int. J. Mod. Phys. D 25 (2016) 1650013.
[26] V. Husain, D. Kothawala, S.S. Seahra, Phys. Rev. D 87 (2013) 025014.
[27] M. Faizal, B. Majumder, Ann. Phys. 357 (2015) 49.
[28] M. Faizal, Int. J. Geom. Methods Mod. Phys. 12 (2015) 1550022.
[29] J.R. Taylor, Scattering Theory, J. Wiley & Sons Inc., NY, 1972.
[30] H.M. Nussenzveig, Causality and Dispersion Relations, Acad. Press, NY-L, 1972.
[31] V.B. Beresteckij, E.M. Lifshitz, L.P. Pitaevskij, Quantum Electrodynamics, Butterworth-Heinemann, NY, 1980.
[32] I.S. Gradshteyn, I.M. Ryzhik, Tables of Integrals, Series, and Products, 7th edition, Academic Press, San Diego U.S.A., 2007.
[33] S.E. Korenblit, A.V. Sinitskaya, Mod. Phys. Lett. A 32 (2017) 1750066.
[34] S.E. Korenblit, D.V. Taychenachev, Mod. Phys. Lett. A 30 (2015) 1550074.
[35] V.A. Naumov, D.S. Shkirmanov, Eur. Phys. J. C 73 (2013) 2627.
[36] D.V. Naumov, V.A. Naumov, D.S. Shkirmanov, Phys. Part. Nucl. 48 (1) (2017) 12.
[37] L.J. Garay, Int. J. Mod. Phys. A 10 (1995) 145.
[38] M. Kober, Phys. Rev. D 82 (2010) 085017.
[39] C. Bambi, F.R. Urban, Class. Quantum Gravity 25 (2008) 095006.
[40] K. Nozari, Phys. Lett. B 629 (2005) 41.
[41] A. Kempf, J. Phys. A 30 (1997) 2093.
[42] A.F. Ali, S. Das, E.C. Vagenas, Phys. Rev. D 84 (2011) 44013.
[43] S. Masood, M. Faizal, Z. Zaz, A.F. Ali, J. Raza, M.B. Shah, Phys. Lett. B 763 (2016) 218.
[44] J. Magueijo, L. Smolin, Phys. Rev. Lett. 88 (2002) 190403.
[45] A.F. Ali, S. Das, E.C. Vagenas, Phys. Lett. B 678 (2009) 497.
[46] J. Magueijo, L. Smolin, Phys. Rev. D 71 (2005) 026010.
[47] P. Horava, Phys. Rev. D 79 (2009) 084008.
[48] P. Horava, Phys. Rev. Lett. 102 (2009) 161301.
[49] G. 't Hooft, Class. Quantum Gravity 13 (1996) 1023.
[50] V.A. Kostelecky, S. Samuel, Phys. Rev. D 39 (1989) 683.
[51] G. Amelino-Camelia, J.R. Ellis, N. Mavromatos, D.V. Nanopoulos, S. Sarkar, Nature 393 (1998) 763.
[52] R. Gambini, J. Pullin, Phys. Rev. D 59 (1999) 124021.
[53] S.M. Carroll, J.A. Harvey, V.A. Kostelecky, C.D. Lane, T. Okamoto, Phys. Rev. Lett. 87 (2001) 141601.
[54] G.B. Whitham, Linear and Nonlinear Waves, John Wiley & Sons, NY, 1999.